ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 31, Number 2, Spring 2001

GENERAL HELICES IN THE THREE-DIMENSIONAL LORENTZIAN SPACE FORMS

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ABSTRACT. We present some Lancret-type theorems for general helices in the three-dimensional Lorentzian space forms which show remarkable differences with regard to the same question in Riemannian space forms. The key point will be the problem of *solving natural equations*. We give a geometric approach to that problem and show that general helices in the three-dimensional Lorentz-Minkowskian space are geodesics either of right general cylinders or of flat B-scrolls. In this sense the anti De Sitter and De Sitter worlds behave as the spherical and hyperbolic space forms, respectively.

1. Introduction. A general helix in the Euclidean space $\mathbf{R}^3$ is a curve which forms a constant angle with a fixed direction in $\mathbf{R}^3$, that is, its tangent indicatrix is a planar curve. The line perpendicular to that plane is called the axis of the general helix. A classical result stated by Lancret in 1802 and first proved by de Saint Venant in 1845 (see [**11**] for details) says that a "curve is a general helix if and only if the ratio of curvature to torsion is constant."

Given a pair of functions, one would like to get an arclength parametrized curve for which that couple works as the curvature and torsion functions. This problem is classically known as the *solving natural equations problem* (see [**11**]). The natural equations for general helices can be integrated in $\mathbf{R}^3$ and in the three-sphere $\mathbf{S}^3$, showing that general helices are geodesics either of right general cylinders or of Hopf cylinders, according to whether the curve lies in $\mathbf{R}^3$ or $\mathbf{S}^3$, respectively (see [**3**] for further details). The hyperbolic space is poor in these kinds of curves because the only general helices are the right circular ones.

2000 AMS *Mathematics Subject Classification.* 53C50, 53A35.
Key words and phrases. General helix, Lorentzian space form, Killing fields along curves, solving natural equations problem.
This paper has been partially supported by DGICYT grant PB97-0784 and Fundación Séneca (CARM) grant PB/5/FS/97.
Received by the editors on March 17, 1999.