

## CONTINUOUS HOMOMORPHISMS BETWEEN TOPOLOGICAL ALGEBRAS OF HOLOMORPHIC GERMS

LUCIANO O. CONDORI AND M. LILIAN LOURENÇO

**ABSTRACT.** We study  $\tau_w$ -continuous homomorphisms on algebras of holomorphic germs. In this setting we give conditions for these homomorphisms to be composition operators. We also present equivalent conditions for the above homomorphisms to be Montel or reflexive.

**1. Introduction.** Let  $E$  be a Banach space,  $U$  an open subset of  $E$  and  $\mathcal{H}(U)$  the space of all holomorphic functions on  $U$ . A holomorphic germ on a compact subset  $K$  of  $E$  is an equivalence class determined in the set of all holomorphic functions on open neighborhoods of  $K$  by the relation,  $f \cong g$  if  $f$  and  $g$  coincide on an open neighborhood of  $K$ . We will denote by  $\mathcal{H}(K)$  the algebra of all holomorphic germs on  $K$ . The natural topology on spaces of holomorphic germs is the Nachbin ported topology  $\tau_w$ . It is defined on  $H(U)$  by the family of all semi-norms ported by the compact subsets of  $U$ . A semi-norm  $p$  on  $H(U)$  is ported by the compact subset  $L$  of  $U$  if for every open subset  $V$ ,  $L \subset V \subset U$ , there is a  $c > 0$  such that  $p(f) \leq c\|f\|_V = c \sup_{x \in V} |f(x)|$  for every  $f \in H(U)$ .

Now, the topology  $\tau_w$  on  $\mathcal{H}(K)$  is the locally convex topology defined by the inductive limit of the spaces  $(H(U), \tau_w)$ , where  $U$  varies over all open neighborhoods of  $K$ . We remark that the space  $(\mathcal{H}(K), \tau_w)$  can be represented as an inductive limit of Banach spaces, namely the inductive limit of the spaces  $H^\infty(U)$ , where  $H^\infty(U)$  denotes the Banach space of all bounded holomorphic functions on  $U$ . We are interested in the study of continuous homomorphisms between locally  $m$ -convex algebras of holomorphic germs. The continuous homomorphisms between topological algebras of holomorphic functions have been extensively studied lately. For example see [4–6]. In [16] Nicodemi has

---

2000 AMS *Mathematics Subject Classification.* Primary 46G20, 32A38, 47B33.  
*Key words and phrases.* Holomorphic germs, continuous homomorphisms.

Research of the first author supported by a doctoral fellowship from CNPq, Brazil.

Received by the editors on October 3, 2003, and in revised form on Feb. 18, 2004.