Fixed point theorems for convex-power condensing operators relative to the weak topology and applications to Volterra integral equations

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Abstract

In this paper we present new fixed point theorems for weakly sequentially continuous mapping which are convex-power condensing relative to a measure of weak noncompactness. Our fixed point results extend and improve several earlier works. As an application, we investigate the existence of weak solutions to a Volterra integral equation.

Key words: Convex-power condensing operators; Fixed point theorems; Measure of weak noncompactness.

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1. Introduction

During the last four decades several interesting studies relating to the existence of weak solutions to the Cauchy differential equation in Banach spaces have been presented. These studies were initiated by Szep [23] in 1971 and since then have been addressed by many investigators. We quote the contributions by Cramer, Lakshmikantham and Mitchell [5] in 1978 and more recently by Bugajewski [4], Cichon [6, 8], Cichon and Kubiaczyk [7], Mitchell and Smith [17], and O’Regan [18, 19, 20]. Motivated by the paper of Cichon [6], D. O’Regan [18] discussed in detail the problem (which was modelled off a first-order differential equation [6])

\[
x(t) = x_0 + \int_0^t f(s, x(s))ds, \quad t \in [0, T];
\]

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