Blow-up Behavior of Hammerstein-type Volterra Integral Equations*

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Abstract

In this paper, we consider the blow-up behavior of Hammerstein-type Volterra integral equations. Based on several fundamental assumptions, some necessary and sufficient conditions under which the solution blows up in finite time are given. Some examples illustrate that there may always exist a global solution for a power-law function and that the blow-up behavior only depends on the value of the kernel in a neighborhood of zero. As an application, we give some results on the blow-up behavior of Volterra integro-differential equation of Hammerstein-type.

2010 Mathematics Subject Classification. Primary: 45D05, 45G10.
Keywords. Volterra integral equations, Volterra integro-differential equations, blow-up, critical exponent.

1 Introduction

In this paper, we investigate the blow-up behaviors of solutions of Hammerstein-type Volterra equations

$$u(t) = \phi(t) + \int_0^t k(t - s)G(s, u(s))ds,$$

(1.1)

*The research leading to this paper has been supported by the Natural Sciences and Engineering Research Council of Canada (NSERC Discovery Grant A9406), the Hong Kong Research Grants Council (RGC Project No. 200210), the National Natural Science Foundation of China (11071050), and the Natural Scientific Research Innovation Foundation at Harbin Institute of Technology (HIT.NSRIF.2010051).