ON AN INTEGRAL EQUATION WITH
A BESSEL FUNCTION KERNEL

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ABSTRACT. The integral equation

\[ g(x) = \int_0^\infty J_x^2(y)f(y)dy, \quad x \geq 0, \]

which arises in certain problems in stereology, is solved for
a wide class of input functions \( g(x) \) using transform tech­
niques. Practical sufficient conditions for the validity of the
solution representation are given and illustrative examples are
presented.

1. Introduction and formal analysis. Integral equations of the
first kind are typically far more difficult to solve than those of the
second kind. Exceptions occur in the case of difference kernels (see,
for example, [2, pp. 301ff.], [9, pp. 364ff], [10]), product or quotient
kernels [3, pp. 214ff], and when transform techniques are applicable.
The problem-at-hand falls into this latter category.

Our interest is in finding the function \( f(y) \) which satisfies the integral
equation

\[ g(x) = \int_0^\infty J_x^2(y)f(y)dy, \quad x \geq 0, \]

where \( g(x) \) is a known function of the nonnegative real variable \( x \) and
\( J_x(y) \) designates the Bessel function of the first kind of order \( x \) and
argument \( y \). The importance of this equation in spatial statistics or
stereology was first brought to the attention of the author by Eugene
Church of DOA [1]. The squared Bessel function kernel and the
appearance of the independent variable in (1.1) are a bit unusual.
Nevertheless, an inversion formula for this integral equation does exist
and can be derived using a procedure based upon analytic function
theory and our-knowledge of two familiar integral transforms.

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