CUTTING AND PASTING INVOLUTIONS

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ABSTRACT. In a recent paper by the author it was shown that the only invariants for $\mathbb{Z}_p$-equivariant cutting and pasting (where $p$ is an odd prime and $\mathbb{Z}_p$ is the cyclic group of order $p$) are oriented $\mathbb{Z}_p$-stratified cobordism and certain Euler characteristic criteria. This paper shows that for certain classes of $\mathbb{Z}_2$-manifolds, i.e., manifolds with involution, there are $\mathbb{Z}_2$-equivariant cutting and pasting results analogous to the odd prime case. An example is given to show that the results for $\mathbb{Z}_p$-manifolds do not strictly carry over to $\mathbb{Z}_2$-manifolds.

1. Introduction. Let $M^n$ and $N^n$ be non-null $n$-dimensional closed smooth (oriented) $\mathbb{Z}_2$-manifolds, i.e., (oriented) manifolds with (orientation preserving) involutions. This paper gives necessary and sufficient conditions under which two $\mathbb{Z}_2$-manifolds $M^n$ and $N^n$ are $\mathbb{Z}_2$-equivariant cut and paste equivalent for certain classes of $\mathbb{Z}_2$-manifolds. The results are analogous to the odd prime case [7], but the results do not strictly carry over for involutions.

It should be noted that the cutting and pasting relation used in this paper is not the same as $SK^*_2$ as found for example in K.K.N.O. [4]. Our cutting and pasting relation has equivariant bordism as an invariant, which is not the case for $SK^*_2$. In Kosniowski's book [5], it is shown that $SK^*_2$ is determined precisely by the Euler characteristics of the manifolds in question, and the Euler characteristics of the fixed sets in each dimension.

We will avoid Kosniowski's language of slice types in equivariant bordism [5]. Instead, we will use a similar notion called "$\mathbb{Z}_2$-stratified cobordism" which was originally defined in [8]. The motivation for the definition of a stratified bordism in the above sense is that it geometrically suggests how one would perform equivariant surgery used in detecting $\mathbb{Z}_2$ cutting and pasting invariants.

DEFINITION 1.1. If $M^n$ and $N^n$ are $n$-dimensional closed smooth...