NONLINEAR FOURTH-ORDER TWO-POINT
BOUNDARY VALUE PROBLEMS

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ABSTRACT. Existence results formulated in terms of upper and lower functions and Nagumo type conditions are presented for fourth order equations and systems of two second order equations with nonlinear boundary constraints.

1. Introduction. This paper deals with the solvability of boundary value problems associated with fourth order differential equations

\[ x^{(4)} = f(t, x, x', x'', x''') \]

as well as with systems of the form

\[ x'' = f_1(t, x, y, x', y'), \]
\[ y'' = f_2(t, x, y, x', y'). \]

The results are inspired by the ones given by Bebernes and Fraker [3] for the boundary value problem

\[ x'' = f(t, x, x'), \]
\[ (x(0), x'(0)) \in S_1, \quad (x(1), x'(1)) \in S_2, \]

where the function \( f \) is supposed to be continuous on \([0, 1] \times \mathbb{R}^2\) and the boundary sets \( S_1 \) and \( S_2 \) are subsets of \( \mathbb{R}^2 \). The main idea in [3] is to investigate the dependence of \( S_1 \) and \( S_2 \) on the a priori bounds in order that the BVP (3), (4) have a solution.

The standard set of conditions ensuring a priori bounds is the following.

(A) *Lower and upper functions.* There exist functions \( \alpha \) and \( \beta \) satisfying on \([0, 1]\) the inequalities \( \alpha \leq \beta \),

\[ \alpha'' \geq f(t, \alpha, \alpha'), \quad \beta'' \leq f(t, \beta, \beta'). \]

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