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SOME REMARKS ON THE DUNFORD-PETTIS PROPERTY

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ABSTRACT. Let A be the disk algebra, Ω be a compact Hausdorff space and μ be a Borel measure on Ω. It is shown that the dual of $C(\Omega, A)$ has the Dunford-Pettis property. This proved in particular that the spaces $L^1(\mu, L^1/H_0^1)$ and $C(\Omega, A)$ have the Dunford-Pettis property.

1. Introduction. Let E be a Banach space, Ω be a compact Hausdorff space and μ be a finite Borel measure on Ω. We denote by $C(\Omega, E)$ the space of all E-valued continuous functions from Ω and for $1 \leq p < \infty$, $L^p(\mu, E)$ stands for the space of all (class of) E-valued p-Bochner integrable functions with its usual norm. A Banach space E is said to have the Dunford-Pettis property if every weakly compact operator with domain E is completely continuous, i.e., takes weakly compact sets into norm compact subsets of the range space. There are several equivalent definitions. The basic result proved by Dunford and Pettis in [**11**] is that the space $L^1(\mu)$ has the Dunford-Pettis property. A. Grothendieck [**12**] initiated the study of Dunford-Pettis property in Banach spaces and showed that $C(K)$-spaces have this property. The Dunford-Pettis property has a rich history; the survey articles by J. Diestel [**8**] and A. Pełczyński [**15**] are excellent sources of information. In [**8**] it was asked if the Dunford-Pettis property can be lifted from a Banach E to $C(\Omega, E)$ or $L^1(\mu, E)$. M. Talagrand [**18**] constructed counterexamples for these questions so the answer is negative in general. There are, however, some positive results. For instance, J. Bourgain showed (among other things) in [**2**] that $C(\Omega, L^1)$ and $L^1(\mu, C(\Omega))$ both have the Dunford-Pettis property; K. Andrews [**1**] proved that if E^* has the Schur property then $L^1(\mu, E)$ has the Dunford-Pettis property. F. Delbaen [**7**] showed that if A is the disc algebra, then $L^1(\mu, A)$ has the Dunford-Pettis property. In [**17**], E. Saab and P. Saab observed that if $\mathcal{A}$ is a C^*-algebra with the Dunford-Pettis property then $C(\Omega, \mathcal{A})$

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