SOME REMARKS ON THE DUNFORD-PETTIS PROPERTY

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ABSTRACT. Let $A$ be the disk algebra, $\Omega$ be a compact Hausdorff space and $\mu$ be a Borel measure on $\Omega$. It is shown that the dual of $C(\Omega, A)$ has the Dunford-Pettis property. This proved in particular that the spaces $L^1(\mu, L^1/H_0^1)$ and $C(\Omega, A)$ have the Dunford-Pettis property.

1. Introduction. Let $E$ be a Banach space, $\Omega$ be a compact Hausdorff space and $\mu$ be a finite Borel measure on $\Omega$. We denote by $C(\Omega, E)$ the space of all $E$-valued continuous functions from $\Omega$ and for $1 \leq p < \infty$, $L^p(\mu, E)$ stands for the space of all (class of) $E$-valued $p$-Bochner integrable functions with its usual norm. A Banach space $E$ is said to have the Dunford-Pettis property if every weakly compact operator with domain $E$ is completely continuous, i.e., takes weakly compact sets into norm compact subsets of the range space. There are several equivalent definitions. The basic result proved by Dunford and Pettis in [11] is that the space $L^1(\mu)$ has the Dunford-Pettis property. A. Grothendieck [12] initiated the study of Dunford-Pettis property in Banach spaces and showed that $C(K)$-spaces have this property. The Dunford-Pettis property has a rich history; the survey articles by J. Diestel [8] and A. Pełczyński [15] are excellent sources of information. In [8] it was asked if the Dunford-Pettis property can be lifted from a Banach $E$ to $C(\Omega, E)$ or $L^1(\mu, E)$. M. Talagrand [18] constructed counterexamples for these questions so the answer is negative in general. There are, however, some positive results. For instance, J. Bourgain showed (among other things) in [2] that $C(\Omega, L^1)$ and $L^1(\mu, C(\Omega))$ both have the Dunford-Pettis property; K. Andrews [1] proved that if $E^*$ has the Schur property then $L^1(\mu, E)$ has the Dunford-Pettis property. F. Delbaen [7] showed that if $A$ is the disc algebra, then $L^1(\mu, A)$ has the Dunford-Pettis property. In [17], E. Saab and P. Saab observed that if $A$ is a $C^*$-algebra with the Dunford-Pettis property then $C(\Omega, A)$