NON-AUTONOMOUS PARABOLIC PROBLEMS WITH SINGULAR INITIAL DATA IN WEIGHTED SPACES

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ABSTRACT. In this paper, we investigate the well-posedness of non-autonomous parabolic equations in weighted space $L^r_{\delta(x)}(\Omega)$, where $\delta(x)$ is the distance to the boundary. We first establish regularity properties of the extension Dirichlet heat semigroup in $L^r_{\delta(x)}(\Omega)$ and then, under some assumptions, we obtain the existence, uniqueness and regularity of the positive solutions of parabolic equations with critical and subcritical nonlinearity term in those spaces.

1. Introduction. The purpose of this paper is to investigate the following nonlinear parabolic equation

\[
\begin{aligned}
&\begin{cases}
    u_t - \Delta u = a(x)u^q + f(x,u) + g(x,t) & t > \tau, \ x \in \Omega, \\
    u = 0 & t > \tau, \ x \in \partial \Omega, \\
    u(\tau) = u_\tau & \tau \in \mathbb{R}, \ x \in \Omega,
\end{cases}
\end{aligned}
\]  

(1.1)

where $\Omega$ is a bounded domain in $\mathbb{R}^N$ with smooth $C^2$ boundary $\partial \Omega$, $0 < q \leq 1$, and $a(x)$, $f(x,u)$ satisfy some conditions which will be stated below.

There has been a great deal of study on problem (1.1) with autonomous cases, i.e., $g(x,t) = 0$. Since the pioneering work of Weissler [16, 17], Ni and Sacks [9], Brezis and Cazenave in [3] studied autonomous problem

\[
\begin{aligned}
&\begin{cases}
    u_t - \Delta u = |u|^{p-1}u & t > 0, \ x \in \Omega, \\
    u = 0 & t > 0, \ x \in \partial \Omega, \\
    u(0) = u_0 & x \in \Omega,
\end{cases}
\end{aligned}
\]  

(1.2)

Keywords and phrases. Parabolic equation, non-autonomous equation, singular initial data, existence, weighted spaces.

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