CYLINDRICAL AND SPHERICAL SOLITONS

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ABSTRACT. In this paper we review briefly the recent prediction and observation of cylindrical and spherical ion acoustic solitons in a cold, single species plasma. The properties of these higher dimensional solitons are quite different from their planar counterpart since the amplitude increases, the width decreases, and the propagation speed increases as the soliton moves inward.

Although the results for two- and three-dimensional solitons are obtained numerically, early time solutions can be found analytically and compared with numerical solutions. An interesting phenomenon, which is an integral part of these higher dimensional solitons, is the presence of a wake. Although the amplitude of this wake is very small, it can carry considerable momentum since it extends over a large spatial region.

1. Introduction. Planar soliton solutions for ion acoustic waves propagating in a collisionless plasma are well known (for a comprehensive review of the subject, see [1]). The initial value problem for the Korteweg-deVries (KdV) equation can be solved numerically, by the inverse method, or by using the conservation laws to find the asymptotic state of $N$ solitons plus continuum. The experimental observation of $N$ soliton breakup and the shape and speed preserving interaction of two solitons is well founded. We remind the reader that the planar soliton is a symmetric pulse moving with constant velocity, for which the square root of the amplitude multiplied by the width is a constant.

Considering the situation where the wave front is curved, and the propagation takes place radially, we expect quite different results. If a solution exists which has support only on a thin cylindrical or spherical shell, the amplitude will have to increase as the radius of the shell gets smaller, so that energy can be conserved. Since the amplitude, width, and propagation speed of soliton solutions are related to one another, we expect these quantities to change considerably as the wave proceeds inward or outward.

2. Spherical Solitons [2]. We assume a collisionless plasma with isothermal electrons and cold ions. A disturbance occurs at some finite radius $r_0$. This causes a perturbation in the ion density $N$ and the elec-


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269